CM03 GRAVITATION & SATELLITES

SPH4U

EQUATIONS

- Law of Universal Gravitation

$$F_G = \frac{Gm_1m_2}{r^2}$$

or

$$F_G = \frac{GMm}{r^2}$$

- Orbital Speed

$$v = \sqrt{\frac{GM}{r}}$$

NEWTON'S LAW OF UNIVERSAL GRAVITATION

Newton's Law of Universal Gravitation

- The force of gravitational attraction between any two objects is directly proportional to the product of the masses of the objects, and inversely proportional to the square of the distance between their centres.

$$F_G = \frac{Gm_1m_2}{r^2}$$

  - G – universal gravitation constant
  - m_1 – mass of first object (usually larger object)
  - m_2 – mass of second object (usually smaller object)
  - r – distance between the centres of the two objects

THINGS TO CONSIDER

- There are two equal, but opposite forces present.
- For the force of attraction to be noticeable, at least one of the objects must be very large.
- Inverse square relationship between F_G and r
  - the force of attraction diminishes rapidly as the two objects move apart
  - there is no value of r, no matter how large, that would reduce the force of attraction to zero
- Every object in the universe exerts a force of attraction on every other object.
- The equation for the law of universal gravitation applies only to:
  - two spherical objects
  - two objects whose sizes are much smaller than their separation distance
  - a small object and a very large sphere

EXAMPLE 1

Earth's gravitational pull on a spacecraft some distance away is 1.2×10^2 N in magnitude. What will the magnitude of the force of gravity be on a second spacecraft with 1.5 times the mass of the first spacecraft, at a distance from Earth's centre that is 0.45 times as great?

EXAMPLE 1 – SOLUTIONS

Let m_E represent the mass of Earth, and the subscripts 1 and 2 represent the first and second spacecraft, respectively.

$F_1 = 1.2 \times 10^2$ N

$m_2 = 1.5m_1$

$r_2 = 0.45r_1$

$F_2 = ?$

By ratio and proportion:

$$\frac{F_2}{F_1} = \frac{\left(\frac{Gm_E m_2}{{r_2}^2}\right)}{\left(\frac{Gm_E m_1}{{r_1}^2}\right)}$$

$$F_2 = F_1\left(\frac{m_2}{{r_2}^2}\right)\left(\frac{{r_1}^2}{m_1}\right)$$

$$= F_1\left(\frac{1.5m_1}{(0.45r_1)^2}\right)\left(\frac{{r_1}^2}{m_1}\right)$$

$$= 1.2 \times 10^2 \text{ N}\left(\frac{1.5}{(0.45)^2}\right)$$

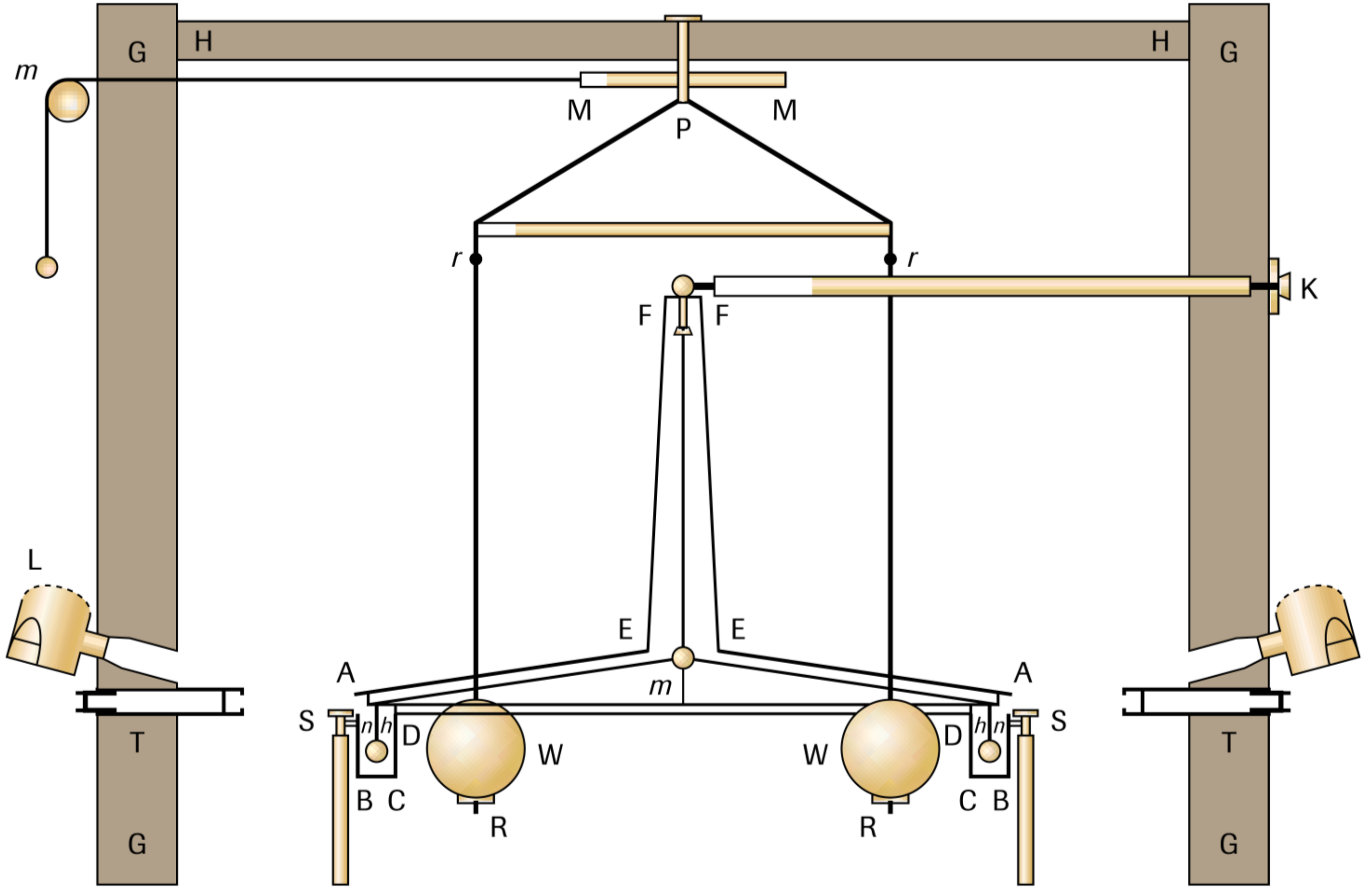
$$F_2 = 8.9 \times 10^2 \text{ N}$$

The force of gravity on the spacecraft is 8.9×10^2 N in magnitude.

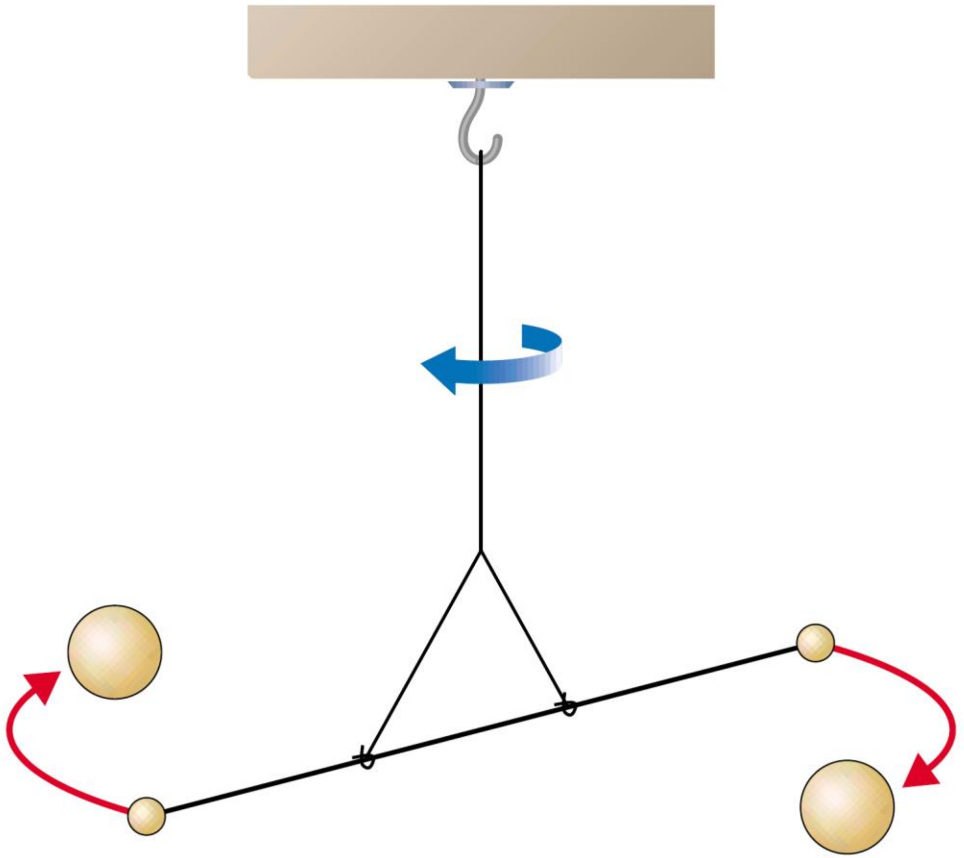
DETERMINING THE UNIVERSAL GRAVITATION CONSTANT

- Over a century after Newton formulated his law, Henry Cavendish succeeded in measuring the gravitational constant
- He found that gravitational force exists even between small objects

HENRY CAVENDISH'S EXPERIMENT



HENRY CAVENDISH'S EXPERIMENT – SIMPLIFIED



EXAMPLE 2

Determine the magnitude of the force of attraction between two uniform metal balls, of mass 4.00 kg, used in women's shot-putting, when the centres are separated by 45.0 cm.

EXAMPLE 2 – SOLUTIONS

$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ $\quad$ $r = 0.450\ \text{m}$

$m_1 = m_2 = 4.00\ \text{kg}$ $\quad$ $F_G = ?$

$$F_G = \frac{Gm_1m_2}{r^2}$$

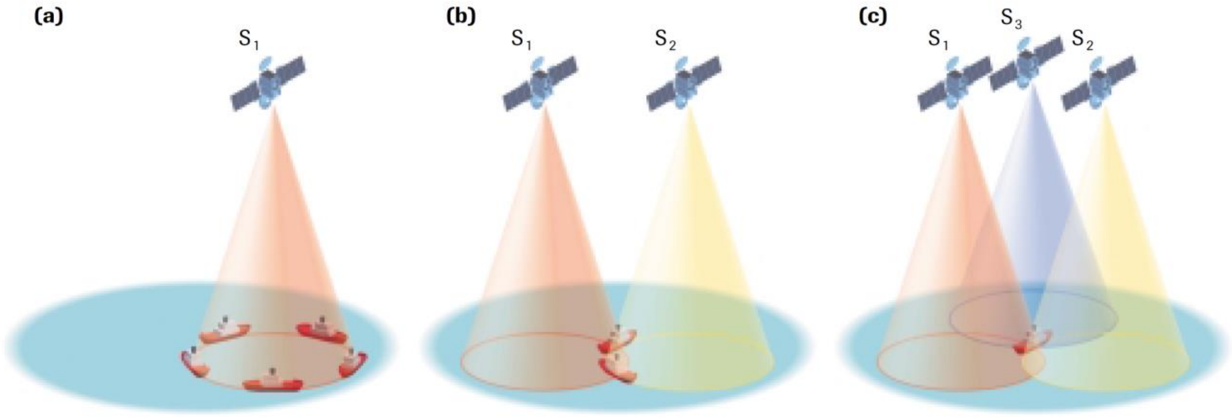
$$= \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4.00\ \text{kg})(4.00\ \text{kg})}{(0.450\ \text{m})^2}$$

$$F_G = 5.27 \times 10^{-9}\ \text{N}$$

The magnitude of the force of attraction is 5.27×10^{-9} N, an extremely small value.

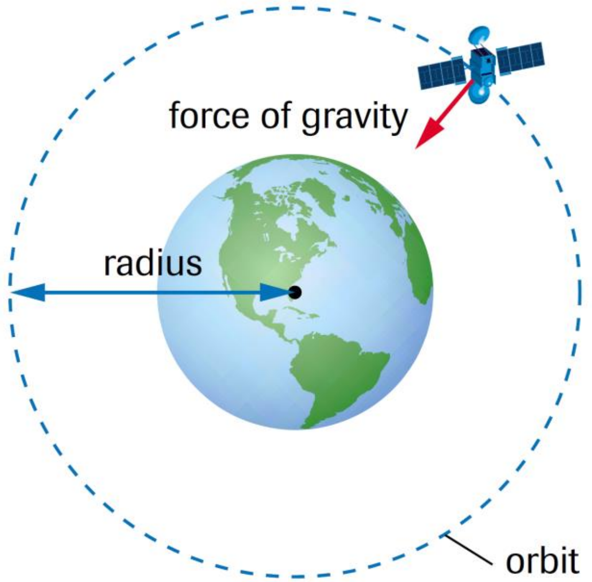
SATELLITES IN CIRCULAR ORBIT

- **Satellite:** object or body that revolves around another body
  - moon around a planet, planets around the Sun
  - artificial satellites (ie: GPS system of 24 satellites)
- **Space Station:** an artificial satellite that can support a human crew and remains in orbit around Earth for long periods



SATELLITES IN CIRCULAR ORBIT – CONT.

- Orbital Radius: the approximate distance a satellite remains from the centre of planet
- A satellite travels fast enough so that as it falls towards Earth, it falls around the planet, maintaining the orbital radius



SPEED OF SATELLITES

- Recall: Centripetal Acceleration and Centripetal Force

$$a_c = \frac{v^2}{r} \qquad \Sigma F = ma_c = \frac{mv^2}{r}$$

- This gives us

$$F_G = \frac{Gm_E m_S}{r^2} = \frac{m_S v^2}{r}$$

  - G – universal constant
  - m_E – mass of the Earth
  - m_S – mass of the satellite
  - v – speed of the satellite
  - r – distance from the centre of the Earth to the satellite

SPEED OF SATELLITES – CONT.

- Solving for the positive value (magnitude of speed):

$$v = \sqrt{\frac{Gm_E}{r}}$$

  - Since G and m_E are constants, v must be constant in order for r to be constant

- **Geosynchronous Satellite:** an satellite with a period of rotation that matches the planet's period of revolution

EXAMPLE 3

The Hubble Space Telescope (HST), shown in **Figure 3**, follows an essentially circular orbit, at an average altitude of 598 km above the surface of Earth.

(a) Determine the speed needed by the HST to maintain its orbit. Express the speed both in metres per second and in kilometres per hour.

(b) What is the orbital period of the HST?

EXAMPLE 3 – SOLUTIONS

(a) $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ $\quad r = 6.38 \times 10^6\ \text{m} + 5.98 \times 10^5\ \text{m} = 6.98 \times 10^6\ \text{m}$

$m_E = 5.98 \times 10^{24}\ \text{kg}$ $\quad v = ?$

Since gravity causes the centripetal acceleration,

$$\frac{Gm_S m_E}{r^2} = \frac{m_S v^2}{r}$$

Solving for v:

$$\begin{aligned} v &= \sqrt{\frac{Gm_E}{r}} \\ &= \sqrt{\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{6.98 \times 10^6\ \text{m}}} \\ &= 7.56 \times 10^3\ \text{m/s} \\ v &= 2.72 \times 10^4\ \text{km/h} \end{aligned}$$

The required speed of the HST is 7.56×10^3 m/s, or 2.72×10^4 km/h.

EXAMPLE 3 – SOLUTIONS CONT.

(b) $v = 2.72 \times 10^4$ km/h

$d = 2\pi r = 2\pi(6.98 \times 10^3 \text{ km})$

$T = ?$

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi(6.98 \times 10^3 \text{ km})}{2.72 \times 10^4 \text{ km/h}}$$

$$T = 1.61 \text{ h}$$

The orbital period of the HST is 1.61 h.

APPARENT WEIGHT

- **Apparent Weight:** the net force exerted on an accelerating object in a noninertial frame of reference

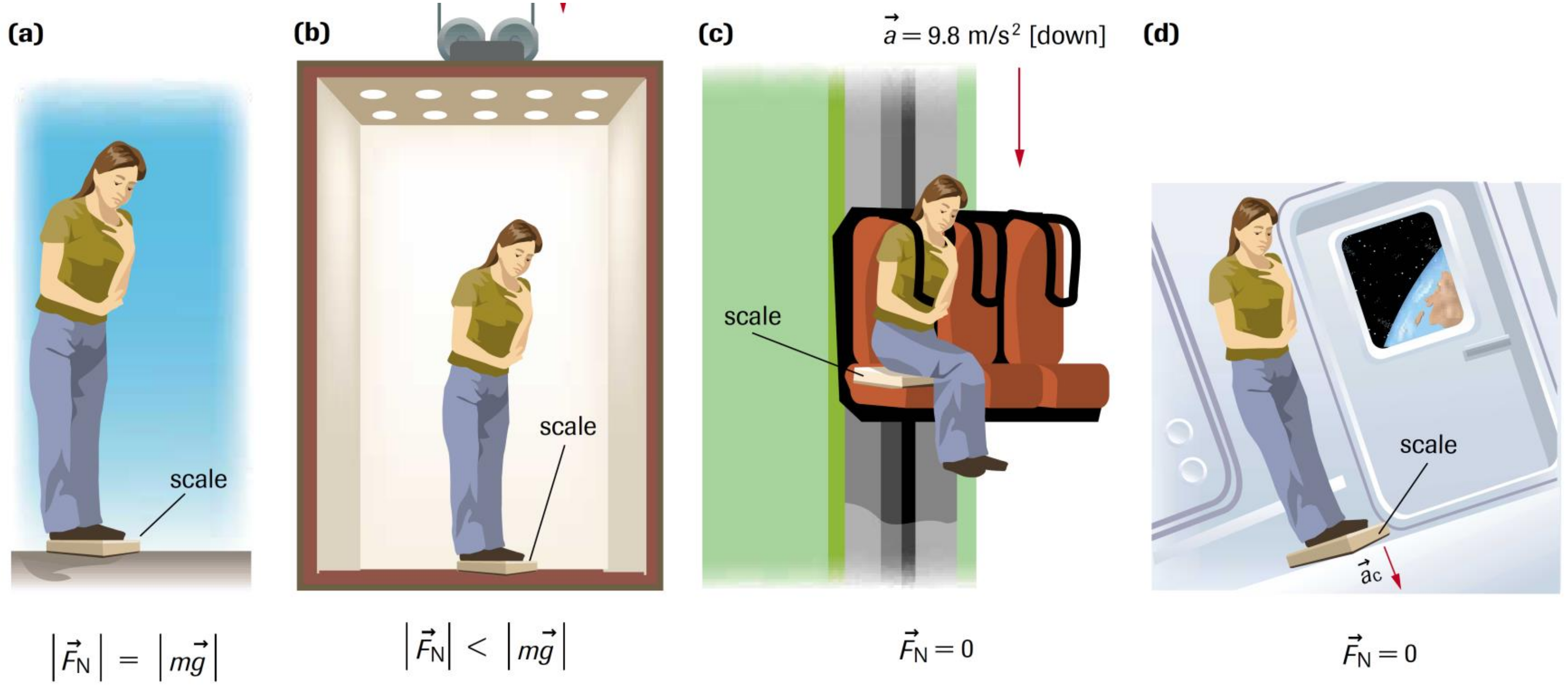
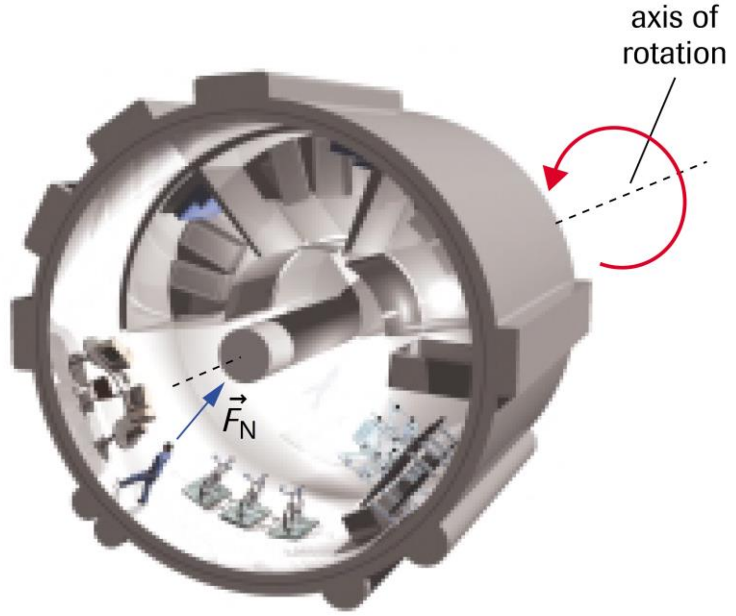


Figure 5
(a) The reading on a bathroom scale is equal to the magnitude of your weight, *mg*.
(b) The reading on the bathroom scale becomes less than *mg* if you weigh yourself on an elevator accelerating downward.
(c) The reading is zero in vertical free fall at an amusement park.
(d) An astronaut in orbit is in free fall, so the reading on the scale is zero.

ARTIFICIAL GRAVITY

- **Artificial Gravity:** situation in which the apparent weight of an object is similar to its weight on Earth
- Usually accomplished by having the spacecraft rotating to create a centripetal force on the astronauts



EXAMPLE 4

You are an astronaut on a rotating space station. Your station has an inside diameter of 3.0 km.

(a) Draw a system diagram and an FBD of your body as you stand on the interior surface of the station.

(b) Determine the speed you need to have if your apparent weight is to be equal in magnitude to your Earth-bound weight.

(c) Determine your frequency of rotation, both in hertz and in revolutions per minute.

EXAMPLE 4 – SOLUTIONS

(a) **Figure 8** contains the required diagrams.

(b) The centripetal acceleration is caused by the normal force of the inside surface of the station on your body. Your weight on Earth is *mg.*

$r = 1.5 \text{ km} = 1.5 \times 10^3 \text{ m}$

$v = ?$

$$\sum F = ma_x$$

$$F_N = ma_c$$

$$F_N = \frac{mv^2}{r}$$

$$mg = \frac{mv^2}{r}$$

$$v^2 = gr$$

$$v = \sqrt{gr}$$

$$= \sqrt{(9.8 \text{ m/s}^2)(1.5 \times 10^3 \text{ m})}$$

$$v = 1.2 \times 10^2 \text{ m/s}$$

Your speed must be 1.2×10^2 m/s.

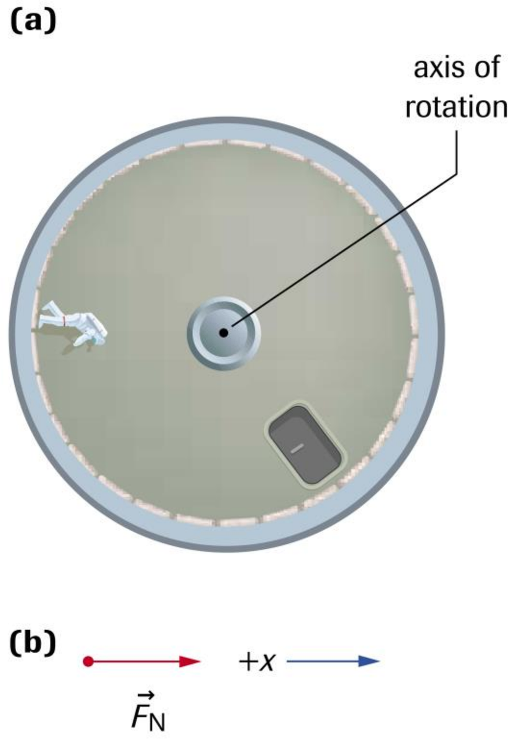


Figure 8
(a) The system diagram of the astronaut and the space station for Sample Problem 2
(b) The FBD of the astronaut

EXAMPLE 4 – SOLUTIONS CONT.

(c) $v = 1.2 \times 10^2$ m/s

$f = ?$

$$v = \frac{2\pi r}{T}$$

$$f = \frac{1}{T}$$

$$v = 2\pi r f$$

$$f = \frac{v}{2\pi r}$$

$$= \frac{1.2 \times 10^2 \text{ m/s}}{2\pi(1.5 \times 10^3 \text{ m})}$$

$$f = 1.3 \times 10^{-2} \text{ Hz, or } 0.77 \text{ rpm}$$

Your frequency of rotation is 1.3×10^{-2} Hz, or 0.77 rpm.

SUMMARY: UNIVERSAL GRAVITATION

- Newton's law of universal gravitation states that the force of gravitational attraction between any two objects is directly proportional to the product of the masses of the objects and inversely proportional to the square of the distance between their centres.
- The universal gravitation constant, $G = 6.67 \times 10^{11}\ \mathrm{Nm^2/kg^2}$, was first determined experimentally by Henry Cavendish in 1798.
- The law of universal gravitation is applied in analyzing the motions of bodies in the universe, such as planets in the solar system. (This analysis can lead to the discovery of other celestial bodies.)

SUMMARY: SATELLITES AND SPACE STATIONS

- Satellites can be natural (such as moons of planets) or artificial (such as the Hubble Space Telescope).
- The speed of a satellite in uniform circular motion around a central body is a function of the mass of that central body and the radius of the orbit. The speed is constant for a given radius.
- Any interplanetary space travel for humans in the future must involve artificial gravity aboard a spacecraft.

PRACTICE

Readings

- Section 3.3 (pg 139)
- Section 3.4 (pg 145)

Questions

- pg 144 #1-3,5,6
- pg 151 #1,2,4-6